On Lag Controllers: Design and Implementation

Marcelo C. M. Teixeira, *Member, IEEE*, and Edvaldo Assunção, *Member, IEEE*

***Abstract*—Sometimes it is inconvenient or expensive to open the loop of a system to insert lag controllers—for instance, when this system is an open-loop system. A new controller structure where the loop is not opened, and that allows the design of lag controllers as in the case where one can open the loop, is presented. This result can be used by educators in undergraduate courses that deal with classic control system theory, because it allows a better comprehension of the concept of lag compensation and provides a new method for its design and implementation. An example illustrates the application of the proposed method.**

***Index Terms*—Classical control, frequency response, lag controllers, root locus, transfer functions.**

I. Introduction

LAG COMPENSATORS, lead compensators, and lead–lag compensators are widely employed for control designers to meet the required performance specifications [1]–[7].

The design of control systems with these controllers is an important topic in many undergraduate engineering courses that deal with classical control system theory—for instance, electrical and electronics engineering, mechanical engineering, mechatronics engineering, and chemical and aerospace engineering courses.

Lag controllers are suitable for the case in which the system exhibits satisfactory transient-response characteristics but unsatisfactory steady-state error [7]. For the system shown in Fig. 1, the procedure for designing lag controllers by root-locus method is shown in [7].

1) Draw the root-locus plot for the uncompensated system whose open-loop transfer function is $G(s)H(s)$ in Fig. 1. Based on transient-response specifications, locate the dominant closed-loop poles on the roots locus.
2) Assume that the transfer function of the lag compensator is as described in Fig. 2. Then the open-loop transfer function of the compensated system becomes $G_c(s)G(s)H(s)$.
3) Evaluate the particular static error constant specified in the problem.
4) Determine the amount of increase in the static error constant necessary to satisfy the specifications.
5) Determine the pole and zero of the lag compensator that produce the necessary increase in the particular static error constant without appreciably altering the original root loci.

Manuscript received December 22, 2001; revised January 24, 2002. This work was supported in part by FAPESP, FUNDUNESP, and CNPq of Brazil.

The authors are with the Department of Electrical Engineering, Faculdade de Engenharia de Ilha Solteira, Universidade Estadual Paulista (UNESP), 15.385-000 Ilha Solteira SP, Brazil.

Publisher Item Identifier S 0018-9359(02)05068-9.

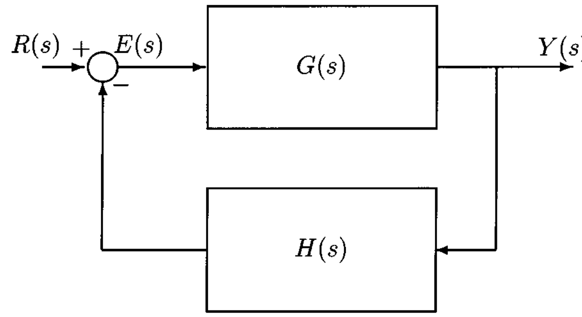

Fig. 1. The system without lag controllers, where $G(s)$ is the plant (and controller) transfer function and $H(s)$ is the sensor transfer function.

6) Draw a new root-locus plot for the compensated system. Locate the desired dominant closed-loop poles on the root locus.
7) Adjust gain K of the compensator from the magnitude condition so that the dominant closed-loop poles lie at the desired location.

Remark 1: The design method proposed in [7] adopted $H(s) = 1$ in Fig. 1. The design steps above are not restricted to $H(s) = 1$.

Note that this design method, based on root locus, assumes that the system has the form described in Fig. 1, and the lag controller is inserted as shown in Fig. 2 [4], [5], [7]. When this system is an open-loop system, the design method above cannot be directly applied. For example, consider a dc servomotor [7] described by the transfer function $W(s)/V(s) = K/(\tau s + 1)$, where $w(t)$ is the rotational velocity, $v(t)$ is the voltage, and K and τ are positive constants. For a step input $v(t) = A$, $t \geq 0$, the motor can present an adequate transient response and an unsatisfactory, steady-state error. In this situation, the motor is an open-loop system, and it is not possible to apply the design method described above. In some cases, it can also be inconvenient or expensive to open the loop to insert the lag controller, as specified in Figs. 1 and 2. An example of this situation can be found in Section III. To avoid this problem, a new control structure for lag controllers is proposed.

II. Design and Implementation of Lag Controllers

Let

$$\frac{Y(s)}{R(s)} = G_{bb}(s)$$

depicted in Fig. 3(a), be a system with satisfactory transient-response characteristics but unsatisfactory steady-state error. This system is equivalent to those shown in Fig. 3(b) and (c). Now,

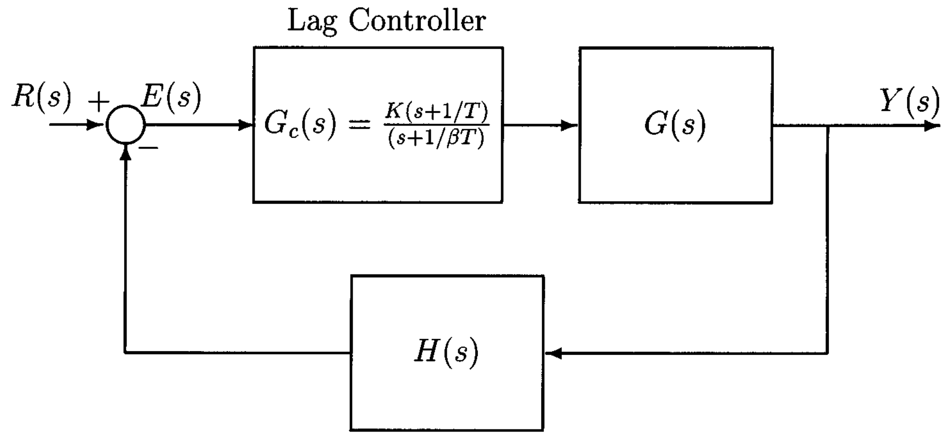

Fig. 2. The usual implementation of lag controllers.

note that the system described in Fig. 3 (c) has the same form as that given in Fig. 1, for

$$H(s)=1 \quad \text{and}$$
$$G(s)=\frac{G_{bb}(s)}{(1-G_{bb}(s))}.$$

Therefore, one can apply the well-known design method for lag controllers using root locus [4], [5], [7], described in Section I, considering

$$G(s)=\frac{G_{bb}(s)}{(1-G_{bb}(s))} \quad \text{and}$$
$$H(s)=1$$

in Fig. 1, because now it is possible to open the loop. The complete controlled system is displayed in Fig. 3(d). The stability of this system [Fig. 3(d)] is guaranteed for $K \approx 1$, $\beta > 1$, and for positive and large values of T.

III. An Application

Consider the transfer function of a closed-loop electrohydraulic servomechanism [4] given in (1) at the bottom of the page. In this case, the designer can access only the input and output of the closed-loop systems (see details in [4]).

The unit step response of this system is shown in Fig. 4(a). Note that the steady-state error is 0.5. Assume that the transient response is satisfactory and that the steady-state error that satisfies the specifications is $e(\infty) = 0.02$ for a unit step input. In this situation, the design method can be applied. Following the method given in Section II, first the authors feedback the system given in Fig. 3(a) twice, as shown in Fig. 3(b). Note that $Y(s)/R(s) = Y(s)/R_N(s)$ in Fig. 3(a) and (b). In Fig. 3(c), the internal transfer function is shown in (2) at the bottom of the page.

Now, the system in Fig. 3(c) is in the adequate form described in Fig. 1, for $H(s) = 1$, and the seven steps described as a procedure for designing lag controllers by root locus method [7], shown in Section I, can be applied. In step 1, the dominant closed-loop poles $-5.4236 \pm 47.2157j$ are found. The static position error for the system in Fig. 1 is $K_p = G(0) = 1$, and therefore the error for a unit step is

$$e(\infty)=\frac{1}{1+K_p}=0.5. \tag{3}$$

In Fig. 4, curve (a), the unit step response of this system is shown.

Now, the specification is $e(\infty) \leq 0.02$, for a unit step input. Therefore, from the steady-state error equation $e(\infty) = 1/(1+K_p)$, it is required that $K_p \geq 49$. Hence, the authors specify $K_p = 50$, and the amount of increase in K_p is $50/1 = 50$. Then, $G_c(0) = K\beta = 50$. The authors adopt $K = 1$ and $\beta = 50$. Finally, they must specify T for $G_c(s) \approx 1\angle 0°$ for s equal to the dominant poles. For $T = 0.1$ and $s = s_o = -5.4236 + 47.2157j$, it follows that

$$G_c(s_0)=\frac{K(s_0+\frac{1}{T})}{(s_0+\frac{1}{\beta T})}=0.99859\angle{-11.849°}.$$

$$G_{bb}(s)=\frac{2.5330\times 10^{-4}s^2+1.5915\times 10^{-3}s+1}{1.0340\times 10^{-5}s^3+9.9761\times 10^{-4}s^2+3.2960\times 10^{-2}s+2}. \tag{1}$$

$$\frac{Y(s)}{E(s)}=G(s)=\frac{G_{bb}(s)}{1-G_{bb}(s)}$$
$$=\frac{2.5330\times 10^{-4}s^2+1.5915\times 10^{-3}s+1.0000}{1.0339\times 10^{-5}s^3+7.4431\times 10^{-4}s^2+3.1368\times 10^{-2}s+1.0000}. \tag{2}$$

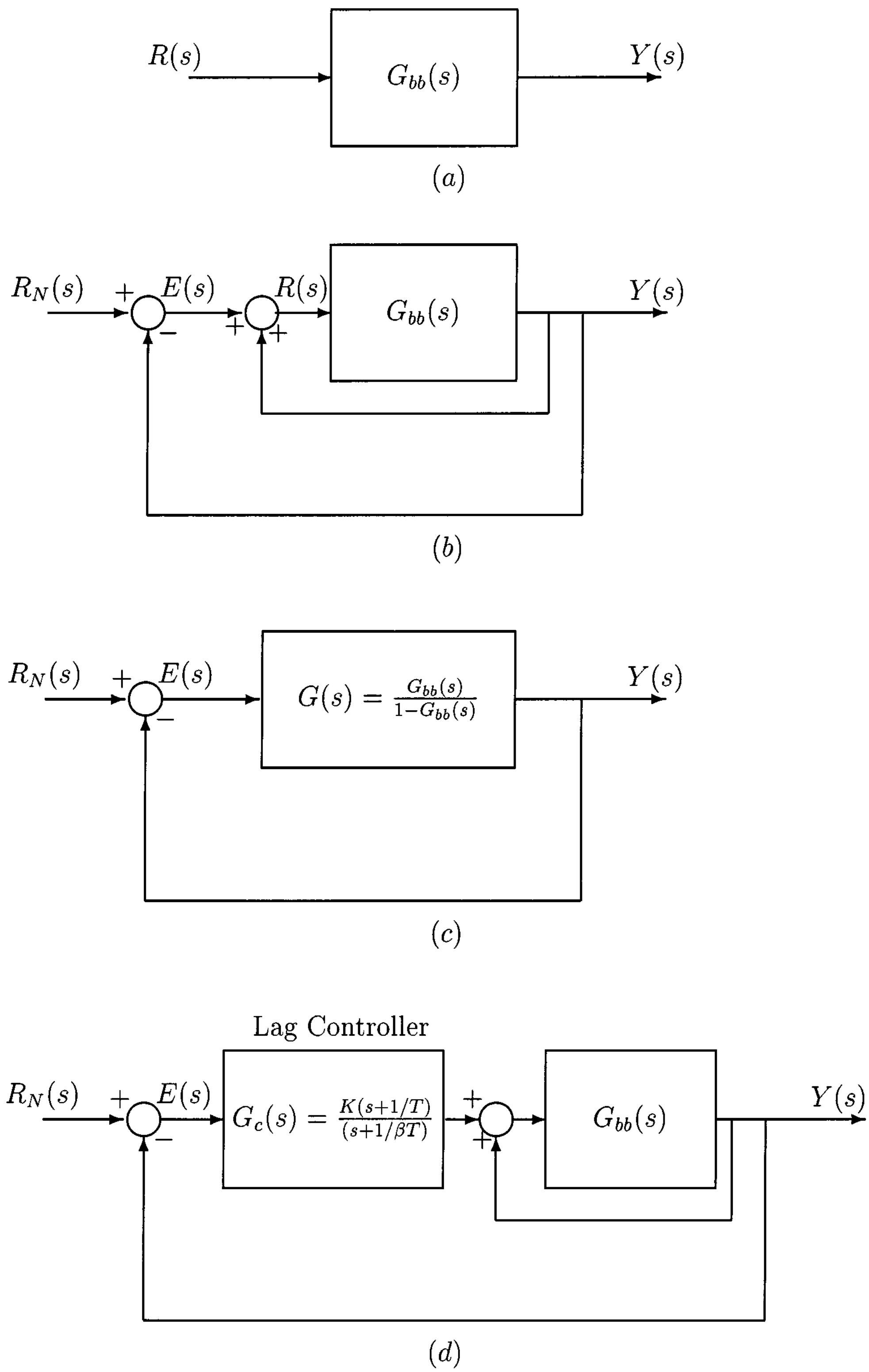

Fig. 3. Description of the proposed method. (a) Original system. (b) Equivalent system. (c) Equivalent system. (d) The proposed implementation of lag controllers.

Although the angle is not so close to 0°, the unit step response shown in Fig. 4, curve (b), is adequate. Therefore, $T = 0.1$ is a convenient choice.

The controlled system is described in Fig. 3(d), where $T = 0.1$, $K = 1$, and $\beta = 50$.

Note that in this example, the authors could use $R(s) = K_0\tilde{R}(s)$ in Fig. 2(a) and specify K_0 for a suitable steady-state error, but the control structure described in Fig. 3(d) is more robust.

IV. Conclusion

A method for the design and implementation of lag controllers without opening the loop and using the well-established

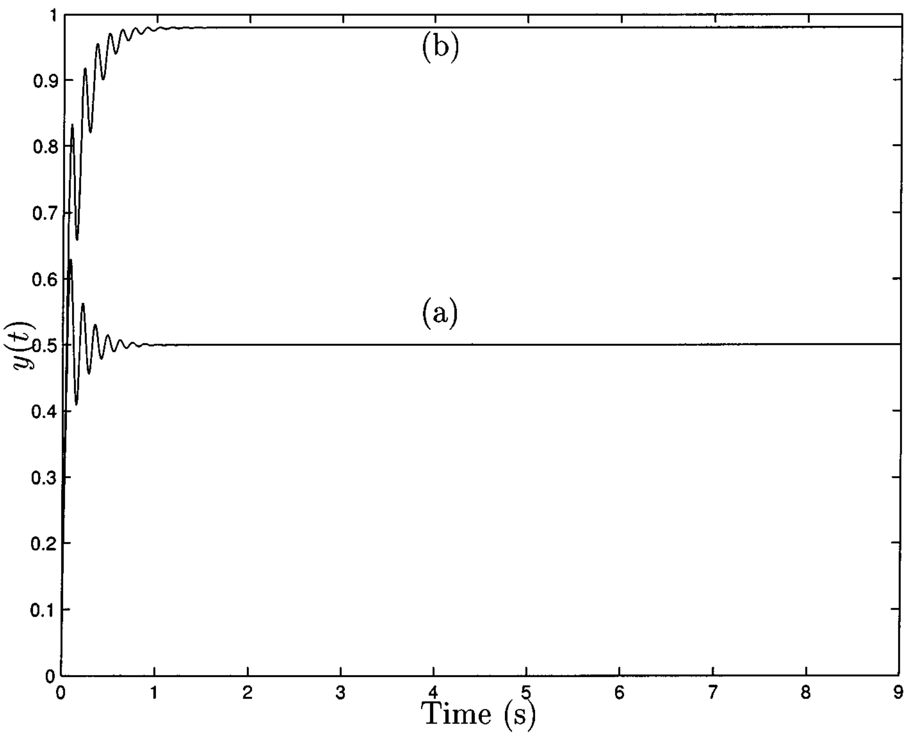

Fig. 4. Unit step responses of an electrohydraulic servomechanism. (a) Original system. (b) Feedback system with $K = 1, T = 0.1$, and $\beta = 50$.

design procedure based on root locus is presented. This method can be useful in designs and implementations of other control systems and can also be adequate in control education [8], [9] because the proposed method is a simple application of block diagram manipulation in the design of the well-known lag controller.

The application of these new results in courses on control systems could be as an example, after the topic about the design of lag controllers based on root locus methods.

The proposed design method and implementation of lag controllers can also be used for lag compensation techniques based on the frequency-domain approach, for example, the design procedure described in [7].

The procedure for designing lead–lag controllers described in [7] and [8] also considers the uncontrolled system described by Fig. 1. Therefore, the ideas given in Section II can be directly applied in the design of lead–lag controllers by root locus method [7], [8] or frequency-domain approach [7], for instance, for open-loop plants.

ACKNOWLEDGMENT

The authors gratefully acknowledge the Editor, the Associate Editor, and the referees for their valuable comments.

REFERENCES

[1] C. T. Chen, *Analog and Digital Control System Design*. Orlando, FL: Saunders College, 1993.
[2] J. J. D'Azzo and C. H. Houpis, *Linear Control System Analysis and Design: Conventional and Modern*. New York: McGraw-Hill, 1995.
[3] J. J. Distefano, A. R. Stubberud, and I. J. Willians, *Theory and Problems of Feedback and Control Systems With Application to the Engineering, Physical and Life Sciences*. New York: McGraw-Hill, 1964.
[4] R. C. Dorf and R. H. Bishop, *Modern Control Systems*. Reading, MA: Addison-Wesley, 1998.
[5] G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*. Reading, MA: Addison-Wesley, 1994.
[6] B. C. Kuo, *Automatic Control System*. Upper Saddle River, NJ: Prentice-Hall, 1995.
[7] K. Ogata, *Modern Control Engineering*. Upper Saddle River, NJ: Prentice-Hall, 1997.
[8] M. C. M. Teixeira, "Direct expressions for Ogata's lead-lag design method using root locus," *IEEE Trans. Educ.*, vol. 37, no. 1, pp. 63–64, 1994.
[9] M. C. M. Teixeira, H. F. Marchesi, and E. Assunção, "Signal-flow graphs: direct method of reduction and MATLAB implementation," *IEEE Trans. Educ.*, vol. 44, pp. 185–190, May 2001.

Marcelo C. M. Teixeira (S'86–M'01) was born in Campo Grande-MS, Brazil, in 1957. He received the B.Sc. degree from Escola de Engenharia de Lins, Brazil, in 1979, the M.Sc. degree from Universidade Federal do Rio de Janeiro, Brazil, in 1982, and the D.Sc. degree from Pontifícia Universidade Católica do Rio de Janeiro, Brazil, in 1989, all in electrical engineering.

In 1982, he joined the Department of Electrical Engineering, Universidade Estadual Paulista (UNESP), Ilha Solteira-SP, Brazil, where he is currently an Associate Professor. In 1996 and 1997, he was a Visiting Scholar in the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN. His interests include control theory and applications, neural networks, variable structure systems, linear matrix inequalities based designs, and fuzzy systems.

Edvaldo Assunção (S'93–M'00) was born in Andradina-SP, Brazil, in 1965. He received the B.Sc. degree from Faculdade de Engenharia de Ilha Solteira (FEIS-UNESP), Brazil, in 1989, the M.Sc. degree from Instituto Tecnológico de Aeronáutica, Brazil, in 1991, and the D.Sc. degree from Universidade Estadual de Campinas, Brazil, in 2000, all in electrical engineering.

In 1992, he joined the Department of Electrical Engineering, UNESP, where he is currently an Assistant Professor. His interests include control theory and applications, model reduction, digital control, linear matrix inequalities based designs, MEMS, and fuzzy systems.

He received the Instituto de Engenharia de São Paulo award from FEIS-UNESP in 1989.